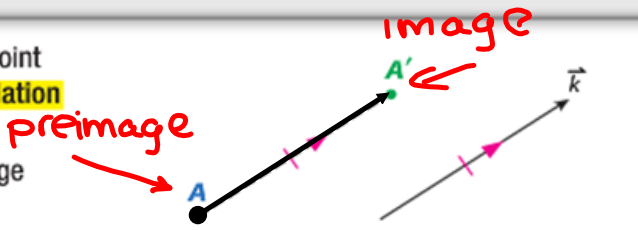
1 Draw Translations In Lesson 4-7, you learned that a translation or *slide* is a transformation that moves all points of a figure the same distance in the same direction. Since vectors can be used to describe both distance and direction, vectors can be used to define translations.

KeyConcept Translation

A translation is a function that maps each point to its image along a vector, called the **translation vector**, such that

- each segment joining a point and its image has the same length as the vector, and
- this segment is also parallel to the vector.



Point A' is a translation of point A along translation vector $\vec{k}$.

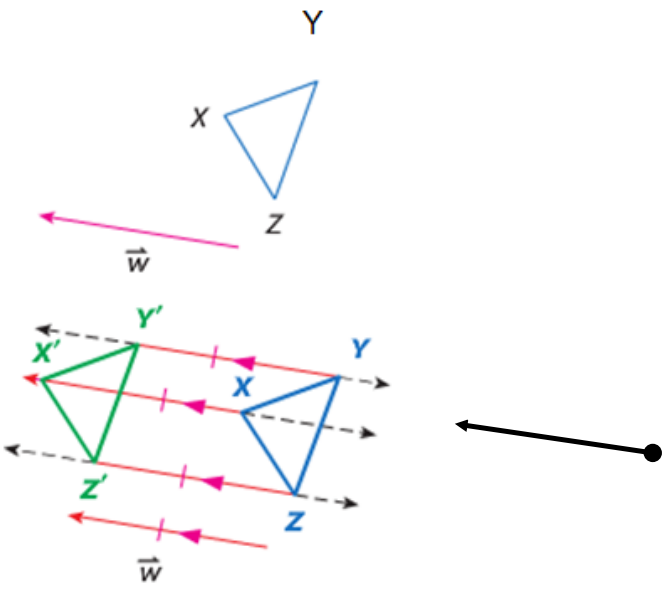
Example 1 Draw a Translation

Copy the figure and the given translation vector. Then draw the translation of the figure along the translation vector.

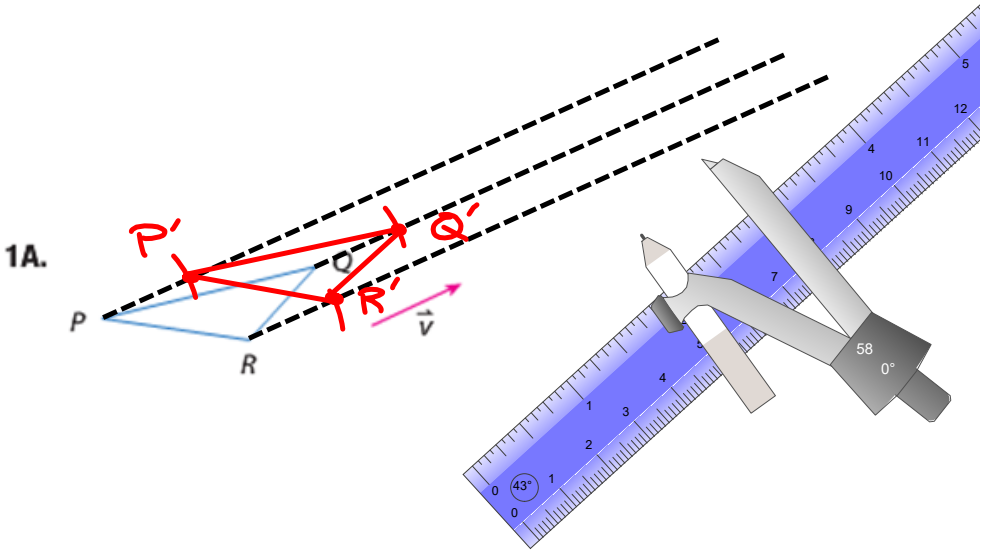
Step 1 Draw a line through each vertex parallel to vector $\vec{w}$

Step 2 Measure the length of vector $\vec{w}$. Locate point X' by marking off this distance along the line through vertex X, starting at X and in the same direction as the vector.

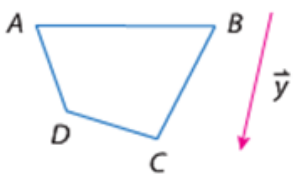
Step 3 Repeat Step 2 to locate points Y' and Z'. Then connect vertices X', Y', and Z' to form the translated image.


Y
X
Z
$\vec{w}$
Y'
X'
Y
X
Z'
Z
$\vec{w}$

1A.



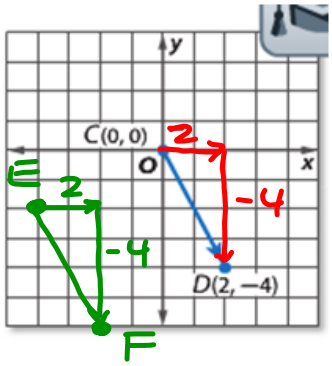
1B.



2 Draw Translations in the Coordinate Plane

Recall that a vector in the coordinate plane can be written as $\langle a, b \rangle$, where a represents the horizontal change and b is the vertical change from the vector's tip to its tail. $\overline{CD}$ is represented by the ordered pair $\langle 2, -4 \rangle$.

Written in this form, called the component form, a vector can be used to translate a figure in the coordinate plane.



a = horizontal ⟷
+a right
-a left

b = vertical change ↕
+b up
-b down

⟨a, b⟩
vector

KeyConcept Translation in the Coordinate Plane

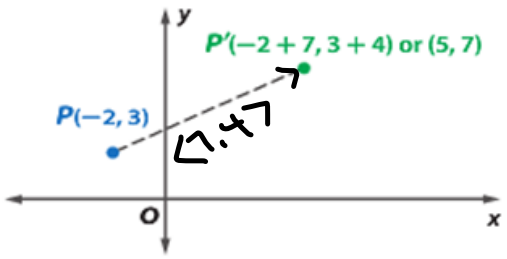
Words To translate a point along vector $\langle a, b \rangle$, add a to the x-coordinate and b to the y-coordinate.

Symbols $(x, y) \rightarrow (x + a, y + b)$

coordinate rule

Example The image of $P(-2, 3)$ translated along vector $\langle 7, 4 \rangle$ is $P'(5, 7)$.

$(-2+7, 3+4)$



A translation is another type of congruence transformation or isometry.

rigid motion

horizontal

only the x-coordinate is affected.

ReadingMath

Horizontal and Vertical Translations When the translation vector is of the form $\langle a, 0 \rangle$, the translation is horizontal only. When the translation vector is of the form $\langle 0, b \rangle$, the translation is vertical only.

only the y-coordinate is affected.

Example 2 Translations in the Coordinate Plane

Graph each figure and its image along the given vector.

a. $\triangle EFG$ with vertices $E(-7, -1)$, $F(-4, -4)$, and $G(-3, -1)$; $\langle 2, 5 \rangle$ → vector

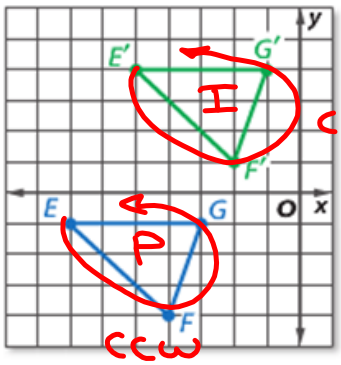
The vector indicates a translation 2 units right and 5 units up.

$(x, y) \rightarrow (x + 2, y + 5)$ coordinate rule

$E(-7, -1) \rightarrow E'(-5, 4)$ = $(-7+2, -1+5)$

$F(-4, -4) \rightarrow F'(-2, 1)$ = $(-4+2, -4+5)$

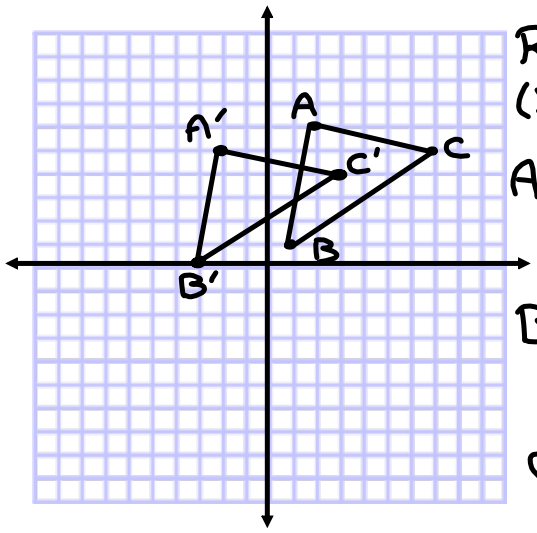
$G(-3, -1) \rightarrow G'(-1, 4)$ = $(-3+2, -1+5)$



This is a direct isometry.

ex. 2

2A. $\triangle ABC$ with vertices $A(2, 6)$, $B(1, 1)$, and $C(7, 5)$ $\langle -4, -1 \rangle$ vector



Rule:

$(x, y) \rightarrow (x-4, y-1)$

$A(2,6) \rightarrow A'(2-4, 6-1)$

$A'(-2, 5)$

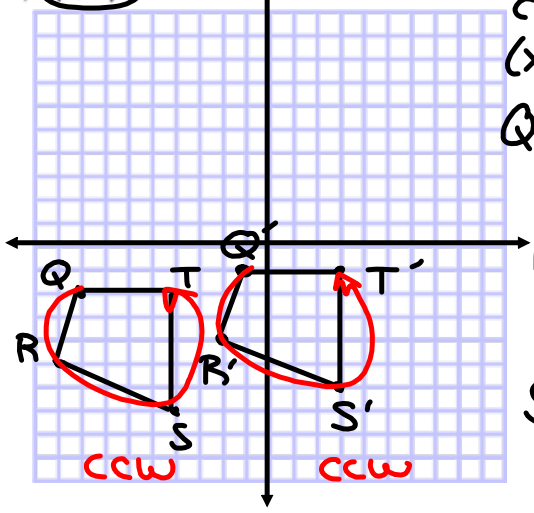
$B(1,1) \rightarrow B'(1-4, 1-1)$

$B'(-3, 0)$

$C(7,5) \rightarrow C'(7-4, 5-1)$

$C'(3, 4)$

2B. quadrilateral $QRST$ with vertices $Q(-8, -2)$, $R(-9, -5)$, $S(-4, -7)$, and $T(-4, -2)$ $\langle 7, 1 \rangle$ vector



direct isometry

coordinate rule:
$(x, y) \rightarrow (x+7, y+1)$

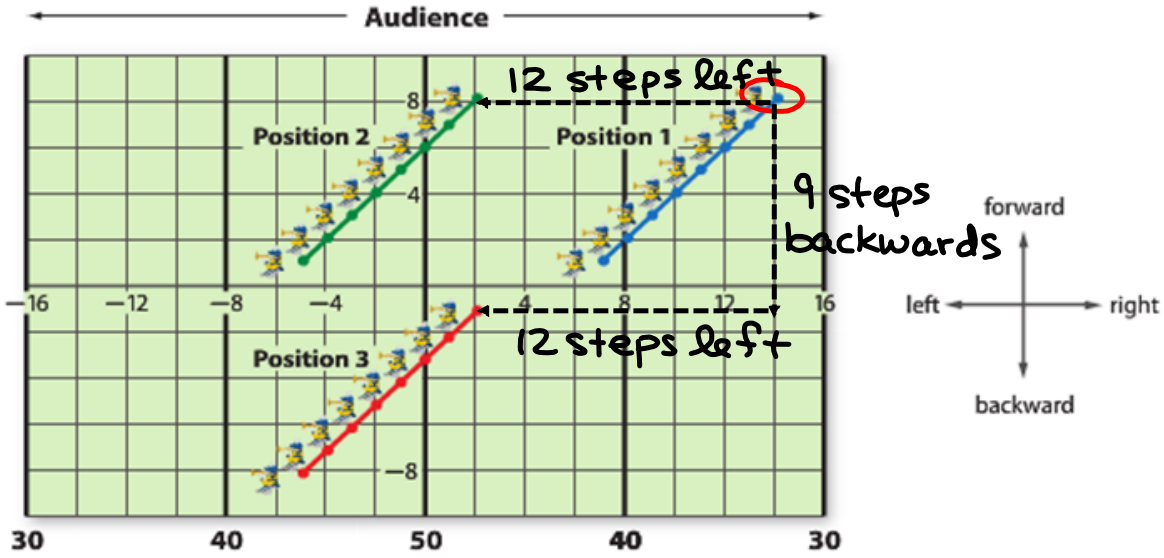
$Q(-8,-2) \rightarrow Q'(-8+7, -2+1)$
$Q'(-1,-1)$

$R(-9,-5) \rightarrow R'(-9+7, -5+1)$
$R'(-2,-4)$

$S(-4,-7) \rightarrow S'(-4+7, -7+1)$
$S'(3,-6)$

$T(-4,-2) \rightarrow T'(-4+7, -2+1)$
$T'(3,-1)$

MARCHING BAND In one part of a marching band's performance, a line of trumpet players starts at position 1, marches to position 2, and then to position 3. Each unit on the graph represents one step.



a. Describe the translation of the trumpet line from position 1 to position 2 in function notation and in words.

function notation: $(x,y) \rightarrow (x-12, y+0)$

words: The trumpet line is translated 12 steps to the left, but no steps forward or backwards.

b. Describe the translation of the line from position 1 to position 3 using a translation vector.

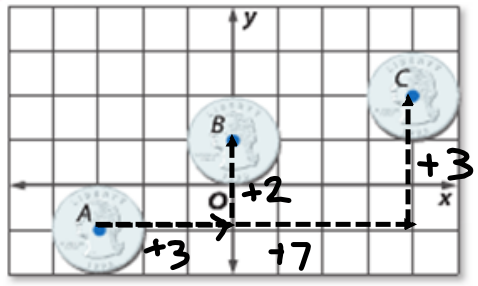
$\langle -12, -9 \rangle$

ex. 3

A coin is filmed using stop-motion animation so that it appears to move.

A. Describe the translation from A to B in function notation and in words.

B. Describe the translation from A to C using a translation vector.



A) notation:

$(x, y) \rightarrow (x+3, y+2)$

words:

Translation 3 units right and 2 units up.

B) $\langle 7, 3 \rangle$